HOMEWORK 1, CALCULUS AND LINEAR ALGEBRA, 2015/2016

**Assigned 09/23/2015, due 09/30/2015, collected from 2pm to 2.15pm sharp!**

**Name and Family Name (CAPITAL LETTERS):** _____

**MATRICOLA N.:** _____

# Exercise 1

Solve the following quadratic equations BY USING THE COMPLETION OF THE SQUARE METHOD:

a) $x^2 + 3x + 2 = 0$

b) $2x^2 - x + \pi = 0$ .

**Solution:**

$$x^2 + 3x + 2 = 0 \iff \left(x + \frac{3}{2}\right)^2 = \frac{1}{4} \iff x + \frac{3}{2} = \pm\frac{1}{2} \iff x = -2 \text{ or } x = -1 .$$

$$2x^2 - x + \pi = 0 \iff \left(x - \frac{1}{4}\right)^2 = -\frac{\pi}{2} + \frac{1}{16} \text{ , no real solutions since } -\frac{\pi}{2} + \frac{1}{16} < 0 .$$

# Exercise 2

Write the equation of the line passing through the points $(1, 1)$ and $(3, 2)$. What is the *angular coefficient* of the line? And its *intersection with the y-axis*? Plot the line on the Cartesian plane.

**Solution:**

$$y - 1 = \frac{2-1}{3-1}(x - 1) \iff y = \frac{1}{2}x + \frac{1}{2} .$$

The angular coefficient is $m = \frac{1}{2}$ and the intercept with the y-axis is $q = \frac{1}{2}$.

# Exercise 3

Write the *even part* and the *odd part* of the following functions:

a) $f(x) = x^4 + 3|x|$

b) $f(x) = x^3 + 1$

c) $f(x) = \frac{x+1}{x^2+1}$ .

**Solution:**

a) $f$ is an even function, hence its even part is $f$ and its odd part is 0.

b) The odd part of $f$ is $x^3$, the even part of $f$ is 1.

c) The even part of $f$ is $\frac{f(x)+f(-x)}{2} = \frac{1}{x^2+1}$, the odd part of $f$ is $\frac{f(x)-f(-x)}{2} = \frac{x}{x^2+1}$.

## Exercise 4

The graph of the function $f(x) = x^2 + 3x + 2$ is a parabola. Find its intersections with the $x$-axis and the $y$-axis. Find the appropriate shifts (horizontal or vertical) such that

a) the shifted parabola intersects the $x$-axis at the points $(-1, 0)$ and $(0, 0)$

b) the shifted parabola intersects the $y$-axis at the point $(0, 0)$ .

**Solution:**

a) To find the points where the parabola $y = f(x)$ intersects the $x$-axis, you have to solve the equation $f(x) = 0$. Following Exercise 1a you find that these points are $(-2, 0)$ and $(-1, 0)$.

Thus you have to shift the parabola on the right by 1. Precisely the shifted parabola is the graph of $f(x - 1) = x^2 + x$.

b) To find the points where the parabola $y = f(x)$ intersects the $y$-axis, one has to compute $y = f(0)$. You find that this point is $(0, 2)$.

Thus you have to shift the parabola down by 2. Precisely the shifted parabola is the graph of $f(x) - 2 = x^2 + 3x$.